RINGS WITH INDECOMPOSABLE RIGHT MODULES LOCAL

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Consider the following condition on a ring R. $(^{*}) : R$ is right artinan and every finitely generated indecomposable right R-module is local.

A dual of this condition is the following. $(^{**}) : R$ is left artinian and every finitely generated indecomposable left module is uniform.

These conditions were first studied by H. Tachikawa in the following two papers:

On rings for which every indecomposable right module has unique maximal submodule, Math. Z., 71 (1959), 200-222.

On algebras over which every indecomposable representation has an irreducible one as the top or the bottom of Leowy constituent, Math. Z., 75 (1961), 215-227.

If a ring R satisfies $(^{*})$, then as shown by Tachikawa, it admits a finitely generated injective cogenerator Q_R and $B = End(Q_R)$ satisfies $(^{**})$. He had studied a ring R satisfying condition $(^{*})$, by the ring B.

Here we study $(^{*})$ without using duality, and determine the structure of local right R-modules. The following result is given in:

S.Singh and H. Al-Bleahed: Rings with indecomposable modules local, Beitrage zur Algebra und Geometrie, 45 (2004), 239-251.

Proposition : Let R be a right artinian ring. R satisfies $(^{*})$ iff the following holds: Let A, B be any two local right R-modules, S, T be simple submodules of A, B respectively. For any R-isomorphism $\sigma : S \to T$, there exists an R-homomorphism η from A to B or from B to A extending σ or σ^{-1}.

Let R be a ring satisfying $(^{*})$, and $J = J(R)$. Let A_R be a local module. It can be proved that AJ contains no local submodule of composition length 3, which is not uniserial, and AJ is a direct sum of uniserial modules. Many results involving extension/lifting of maps between local R-modules are proved. Using these one gets the following.

Theorem : Let R satisfy $(^{*})$. Let (S_R, n, T_R) be a triple, where S, T are simple, semi-simple modules respecctively, and n is a positive integer. Then there do not exist more than two local modules A over R such that $S \cong A/AJ$, $T \cong socle(A)$ and n the composition length of A.

Using the above theorem, it follows that any ring R satisfying $(^{*})$ is of finite representation type. An elementary proof is given to show that any module over R is a direct sum of local modules.